Problem 1

In a problem involving vibrations there will always be some mechanisms that prevents a vibration to continue at the initial level for ever, i.e. there must be other forces present, besides inertia forces ($-m\ddot{u}$) and stiffness ($-ku$),as assumed in problem 1. There will always be some damping, for the pendulum in excercise 1 this will mostly be due to air resistance. The simplest model for damping is to assume a force acting against the motion, equal to ($-c\dot{u}$). I.e. this force acts toward the left when the motion is toward the right. Let us also include a harmonic force as follows

$$f = \mathrm{Re}\left\{ f_0 e^{i\omega t} \right\}$$

a. Find the differential equation for this pendulum in case *a*, including the effects of mass, stiffness, damping and external loading. Also please find a formula for the amplitude of the vibrations, and for the phase angle between the loading and the motion. Find the expressions for the particular and homogenous solutions to obtain the total solution, that is the response with the initial transient phase and the stationary phase.

b. It is observed that when there is no loading the vibration amplitudes reduce by a factor 0.99 from one vibration maximum to the next one. Calculate the damping ratio ξ and express the damping constant *c* in terms of ξ.

c. Now the same system is subjected to a harmonic loading, i.e. $f_0>0$, see above. Set g=9.81 m/s, l=10 m and find (undamped) natural frequency (ω_e), damped natural frequency (ω_d) and the resonant frequency (ω_r) for this case. Also please find the maximum of the dynamic amplification factor $D(\omega)$.

d. Given a mass of 10 kg and the input in the above tasks, consider the maximum amplitude f_0 of the force that can be applied given the small angle approximation made when the differential equation was derived. Also consider for which values of the initial conditions in the homogenous solution the approximation is still valid. Discuss how much error in the calculation of the angle of the pendulum you would accept. For one acceptable combination of initial conditions and *f0* plot the total solution for one case where $\omega<\omega_e$, one where $\omega>\omega_e$ and one case where ω is close to ω_e. Also plot one case where the initial conditions will give a reduction of initial amplitude to the amplitude of the particular solution and one where there will be an increase from initial conditions to the amplitude of the particular solution. It might be useful to look at these solutions with a higher damping than the one in task b, otherwise the time series will be very long.

Problem 2

Consider Example 2.10 in the lecture notes by Naess. Familiarize yourself with the development of the expressions (a) and (b) and refer this in your own words (briefly).

Substitute the usual expressions

$$x = \mathrm{Re}\left\{ x_0 e^{i\omega t} \right\}$$

$$y = \mathrm{Re}\left\{ x_0 H_{xy}(\omega)\, e^{i\omega t} \right\}$$

in the differential equation (c), and find from that $H_{xy}(\omega)$ which as you know is denoted 'the transfer function' from (input) *x* to (output) *y*

Then use the relationship $y=u-x$ to find the transfer function from *x* to *u* and check that you obtain the same result as according to formula (f) in the lecture notes. Now write down the transfer function $H_{xu}(\omega)$.